Polynomials in Matlab

Polynomials

- $f(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0$
- n is the degree of the polynomial
- Examples:

  $f(x) = 2x^2 - 4x + 10$ degree 2

  $f(x) = 6$ degree 0

Polynomials in Matlab

- Represented by a row vector in which the elements are the coefficients.
- Must include all coefficients, even if 0
- Examples

  $8x + 5$ p = [8 5]

  $6x^2 - 150$ h = [6 0 -150]

Value of a Polynomial

- Recall that we can compute the value of a polynomial for any value of x directly.
- Ex: $f(x) = 5x^3 + 6x^2 - 7x + 3$

```
x = 2;
y = (5 * x ^ 3) + (6 * x ^ 2) - (7 * x) + 3
y =
    53
```

Value of a Polynomial

- Matlab can also compute the value of a polynomial at point x using a function, which is sometimes more convenient
- polyval (p, x)
  - p is a vector with the coefficients of the polynomial
  - x is a number, variable or expression

Value of a polynomial

- Ex: $f(x) = 5x^3 + 6x^2 - 7x + 3$

```
p = [5 6 -7 3];
x = 2;
y = polyval(p, x)
y =
    53
```

Roots of a Polynomial

- Recall that the roots of a polynomial are the values of the argument for which the polynomial is equal to zero
- Ex: $f(x) = x^2 - 2x - 3$

$0 = x^2 - 2x - 3$

$0 = (x + 1)(x - 3)$

$0 = x + 1$ OR $0 = x - 3$

$x = -1$ $x = 3$

Roots of a Polynomial

- Matlab can compute the roots of a function
- r = roots(p)
  - p is a row vector with the coefficients of the polynomial
  - r is a column vector with the roots of the polynomial

Roots of a Polynomial

- Ex: $f(x) = x^2 - 2x - 3$

```
p = [1 -2 -3];
r = roots(p)
r =
    3.0000
   -1.0000
```

Polynomial Coefficients

- Given the roots of a polynomial, the polynomial itself can be calculated
- Ex: roots = -3, +2

x = -3 OR x = 2

0 = x + 3 0 = x - 2

0 = (x + 3)(x - 2)

$f(x) = x^2 + x - 6$

Polynomial Coefficients

- Given the roots of a polynomial, Matlab can compute the coefficients
- p = poly(r)
  - r is a row or column vector with the roots of the polynomial
  - p is a row vector with the coefficients

Polynomial Coefficients

- Ex: roots = -3, +2

```
r = [-3 +2];
p = poly(r)
p =
    1    1    -6
% f(x) = x^2 + x -6
```

Adding and Subtracting Polynomials

- Polynomials can be added or subtracted
- Ex: f1(x) + f2(x)

f1(x) = $3x^6 + 15x^5 - 10x^3 - 3x^2 + 15x - 40$

f2(x) = $\underline{\qquad\qquad\qquad 3x^3 \qquad\qquad - 2x - 6}$

$3x^6 + 15x^5 - 7x^3 - 3x^2 + 13x - 46$

Adding and Subtracting Polynomials

- Can do this in Matlab by just adding or subtracting the coefficient vectors
  - Both vectors must be of the same size, so the shorter vector must be padded with zeros

Adding and Subtracting Polynomials

Ex:

f1(x) = $3x^6 + 15x^5 - 10x^3 - 3x^2 + 15x - 40$

f2(x) = $3x^3 - 2x - 6$

```
p1 = [3 15 0 -10 -3 15 -40];
p2 = [0  0  0  3 0 -2 -6];
p = p1+p2
p =
     3    15     0    -7    -3    13   -46
```

%f(x) = $3x^6 + 15x^5 - 7x^3 - 3x^2 + 13x - 46$

Multiplying Polynomials

- Polynomials can be multiplied:
- Ex: $(2x^2 + x - 3) * (x + 1) =$

$$
\begin{array}{lrrrr}
 & 2x^3 & + x^2 & - 3x & \\
\underline{+} & & \underline{2x^2} & \underline{+ x} & \underline{-3} \\
 & 2x^3 & + 3x^2 & - 2x & - 3
\end{array}
$$

Multiplying Polynomials

- Matlab can also multiply polynomials
- c = conv(a, b)
  - a and b are the vectors of the coefficients of the functions being multiplied
  - c is the vector of the coefficients of the product

Multiplying Polynomials

- Ex: $(2x^2 + x - 3) * (x + 1)$

```
a = [2 1 -3];
b = [1 1];
c = conv(a, b)
c =
     2     3    -2    -3
```

% $2x^3 + 3x^2 - 2x - 3$

Dividing Polynomials

- Recall that polynomials can also be divided

```
         x -10
x + 1 | x^2 - 9x - 10
       -x^2 -  x
           -10x -10
           -10x -10
                 0
```

Dividing Polynomials

- Matlab can also divide polynomials
- [q,r] = deconv(u, v)
  - u is the coefficient vector of the numerator
  - v is the coefficient vector of the denominator
  - q is the coefficient vector of the quotient
  - r is the coefficient vector of the remainder

Dividing Polynomials

- Ex: $(x^2 - 9x - 10) \div (x + 1)$

```
u = [1 -9 -10];
v = [1 1];
[q, r] = deconv(u, v)
q =
     1   -10     %  quotient is (x - 10)
r =
     0     0     0   % remainder is 0
```

Example 1

- Write a program to calculate when an object thrown straight up will hit the ground. The equation is

  $s(t) = -½gt^2 + v_0t + h_0$

  s is the position at time t (a position of zero means that the object has hit the ground)

  g is the acceleration of gravity: $9.8 m/s^2$

  v_0 is the initial velocity in m/s

  h_0 is the initial height in meters (ground level is 0, a positive height means that the object was thrown from a raised platform)

Example 1

Prompt for and read in initial velocity

Prompt for and read in initial height

Find the roots of the equation

Solution is the positive root

Display solution

Example 1

```
v = input('Please enter initial velocity:  ');
h = input('Please enter initial height:  ');
x = [-4.9 v h];
y = roots(x);
if y(1) >= 0
   fprintf('The object will hit the ground in %.2f seconds\n',
        y(1))
else
   fprintf('The object will hit the ground in %.2f seconds\n',
        y(2))
end
```

Example 1

Please enter initial velocity: 19.6
Please enter initial height: 58.8
The object will hit the ground in 6.00 seconds

Derivatives of Polynomials

- We can take the derivative of polynomials

$f(x) = 3x^2 - 2x + 4$

$\frac{dy}{dx} = 6x - 2$

Derivatives of Polynomials

- Matlab can also calculate the derivatives of polynomials
- k = polyder(p)
  p is the coefficient vector of the polynomial
  k is the coefficient vector of the derivative

Derivatives of Polynomials

- Ex: $f(x) = 3x^2 - 2x + 4$

```
p = [3 -2 4];
k = polyder(p)
k =
      6    -2
% dy/dx = 6x - 2
```

Integrals of Polynomials

- ?6x^2 dx = 6 ?x^2 dx
  = 6 * ? x^3
  = 2 x^3

Integrals of Polynomials

- Matlab can also calculate the integral of a polynomial

  g = polyint(h, k)

  h is the coefficient vector of the polynomial

  g is the coefficient vector of the integral

  k is the constant of integration - assumed to be 0 if not present

Integration of Polynomials

- ?$6x^2$ dx

```
h = [6 0 0];
g = polyint(h)
g =
     2     0     0     0
% g(x) = 2x^3
```

Polynomial Curve Fitting

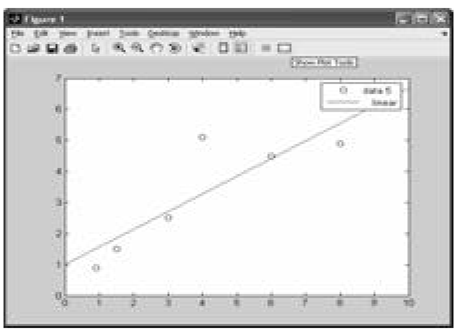
- Curve fitting is fitting a function to a set of data points
- That function can then be used for various mathematical analyses
- Curve fitting can be tricky, as there are many possible functions and coefficients

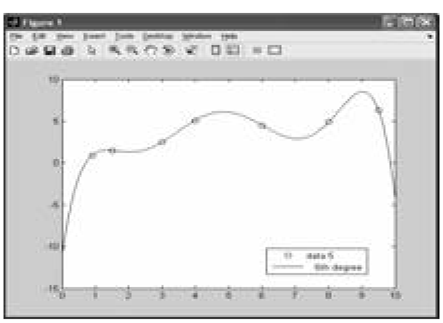
Curve Fitting

- Polynomial curve fitting uses the method of least squares
  - Determine the difference between each data point and the point on the curve being fitted, called the residual
  - Minimize the sum of the squares of all of the residuals to determine the best fit

Curve Fitting

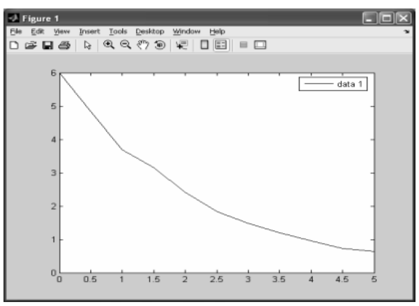
- A best-fit curve may not pass through any actual data points
- A high-order polynomial may pass through all the points, but the line may deviate from the trend of the data

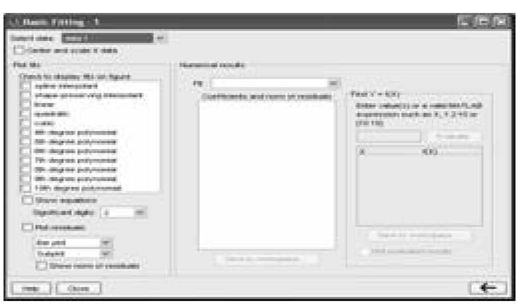




Matlab Curve Fitting

- Matlab provides an excellent polynomial curve fitting interface
- First, plot the data that you want to fit
  t = [0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0];
  w = [6.00 4.83 3.70 3.15 2.41 1.83 1.49 1.21 0.96 0.73 0.64];
  plot(t, w)
- Choose Tools/Basic Fitting from the menu on the top of the plot

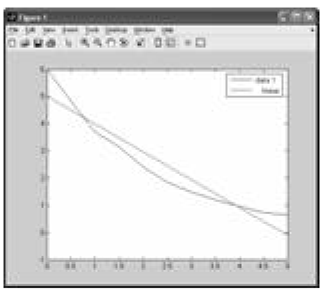




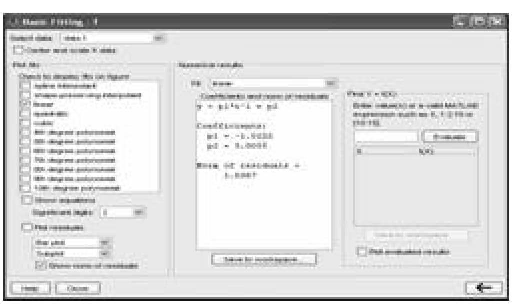
Matlab Curve Fitting

- Choose a fit
  - it will be displayed on the plot
  - the numerical results will show the equation and the coefficients
  - the norm of residuals is a measure of the quality of the fit. A smaller residual is a better fit.
- Repeat until you find the curve with the best fit

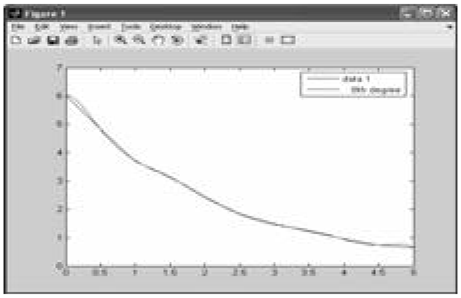
Linear Fit



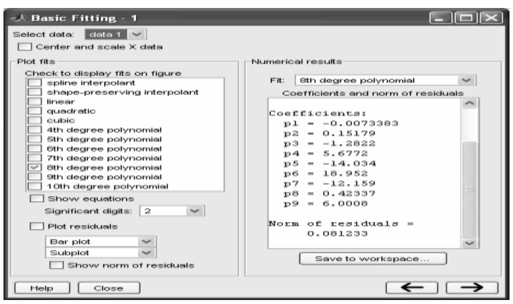
Linear Fit



8th Degree Polynomial



8th Degree Polynomial



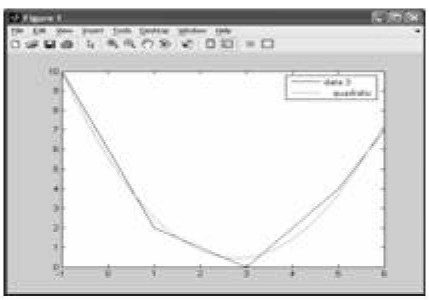
Example 2

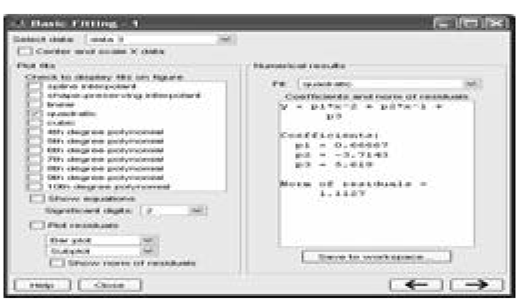
- Find the parabola that best fits the data points (-1, 10) (0, 6) (1, 2) (2, 1) (3, 0) (4, 2) (5, 4) and (6, 7)
- The equation for a parabola is

  $f(x) = ax^2 + bx + a$

Example 2

X = [-1, 0, 1, 2, 3, 4, 5, 6];
Y= [10, 6, 2, 1, 0, 2, 4, 7];
plot (X, Y)





Other curves

- All previous examples use polynomial curves. However, the best fit curve may also be power, exponential, logarithmic or reciprocal
- See your textbook for information on fitting data to these types of curves

Interpolation

- Interpolation is estimating values between data points
- Simplest way is to assume a line between each pair of points
- Can also assume a quadratic or cubic polynomial curve connects each pair of points

Interpolation

- yi = interp1(x, y, xi, 'method')

interp1 - last character is one

x is vector with x points

y is a vector with y points

xi is the x coordinate of the point to be interpolated

yi is the y coordinate of the point being interpolated

Interpolation

- method is optional:
  'nearest' - returns y value of the data point that is nearest to the interpolated x point
  'linear' - assume linear curve between each two points (default)
  'spline' - assumes a cubic curve between each two points

Interpolation

- Example:

```
x = [0 1 2 3 4 5];
y = [1.0 -0.6 -1.4 3.2 -0.7 -6.4];
yi = interp1(x, y, 1.5, 'linear')
yi =
     -1
yj = interp1(x, y, 1.5, 'spline')
yj =
      -1.7817
```